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THE ORBIT DETERMINATION TOOL KIT (ODTK) - VERSION 5

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The *Orbit Determination Tool Kit* (ODTK) is commercial software for geocentric spacecraft orbit determination and analysis from Analytical Graphics, Inc. Its key features include a tracking-data simulator, an optimal (Kalman-like) sequential filter, and a sequential fixed-interval smoother. Functionality includes autonomous measurement editing, customizable reporting and graphing, simultaneous multiple-satellite estimation, and modeling of time-varying system biases. ODTK Version 5 adds two significant capabilities: a broad operational capability for maintaining global navigation satellite systems such as the US Global Positioning System, and the ability to refine location estimates for poorly known emission sources using time difference-of-arrival and frequency difference-of-arrival measurements.

BACKGROUND

Analytical Graphics, Inc. (AGI) provides a commercial software product known as the *Orbit Determination Tool Kit*, or ODTK, for geocentric spacecraft orbit determination and analysis. ODTK employs an established algorithm for sequential orbit determination, first published by Wright (1981) and used by the General Electric Company for satellite operations soon thereafter.[1] During the next twenty years, this algorithm was recoded with little change into a variety of software packages by various companies and agencies, to support several operational satellites.

In 1999, AGI concluded that this highly utilized algorithm would be the ideal basis for its own emerging orbit determination capability. Development of ODTK began under the project codename "Mach 10" before it was introduced as the commercial-off-the-shelf product *STK/OD* in October, 2003. Although STK/OD used many of the same verified astrodynamic functions as AGI's *Satellite Tool Kit* (STK) software, it was not a sub-module of STK and was developed and maintained separately as a stand-alone product. With the fourth major version release, the orbit determination software product was renamed ODTK.

The existing customer base for ODTK is varied, and includes aerospace industry, national governments, academia, and others authorized to use this technology under US export control regulations.[2] Key features of ODTK include a tracking-data simulator, an optimal (Kalman-like) sequential filter, and a matching optimal sequential smoother. ODTK also provides initial orbit determination capability, and a conventional weighted least-squares estimator. The analyst is able to automate ODTK operation with scripts and

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create his own custom “plug-in” models. A wide variety of output data are available for reporting and graphing, using built-in templates or templates created by the analyst.

ODTK’s filter/smoother can estimate the orbits of multiple satellites, their ballistic coefficients and/or solar radiation pressure coefficients, adjust the uncertain priors in ground-based and space-based tracking facilities, calibrate maneuvers, and solve for time-varying measurement biases, transponder biases, and atmospheric density corrections - all simultaneously. This is relevant to missions that combine data from multiple satellites or tracking systems, such as global navigation satellite systems (GNSS), formation flying, geodesy, geolocation, cross-linked tracking, satellite-to-satellite tracking, and intelligence activities where the relative errors between ephemerides are usually important. Given sufficient tracking data, ODTK’s filter can successfully recover the orbits of maneuvering satellites.

With the latest release, ODTK provides capability to estimate a wide range of states parameters and to support a wide variety of missions. This version is enhanced to provide orbit determination for the mission control segment of US Global Positioning System (GPS). Given sufficient tracking data, ODTK can estimate GPS satellite orbit states and associated solar radiation pressure coefficients, receiver antenna locations, receiver clock offsets, and satellite-clocks offsets over the entire constellation - all simultaneously. ODTK can also estimate the location of radio sources on the ground simultaneously with orbit states for transponding satellites with the introduction of *difference-of-arrival* measurements.

Table 1. COMPUTING SYSTEM REQUIREMENTS

	Minimum	**Recommended**
Random-Access Memory	256 MB	512 MB
Disk Space	1 GB	4 GB
Processor	800 MHz	2.0 GHz
Hard Disk Memory	300 MB	900 MB
Operating System	Windows 2000/XP and higher	

Vector processing of simultaneous measurements is added to the most recent version of ODTK for faster performance. Computer system requirements for ODTK are described in Table 1.

USAGE AND INTERFACE

ODTK’s graphical user interface (GUI) has an on-screen *object manager* whose contents parallel other orbit determination processors that use a tabular means to invoke software options (*e.g.*, input “deck”). As object-oriented software, ODTK inputs are described as *objects* having changeable characteristics called *attributes*. Most objects and attributes will be familiar to an experienced orbit determination analyst. An analyst manages (adds, removes, or changes) object options via the object manager, then saves his settings to a *scenario* file. ODTK also provides a *message viewer* window that displays errors, warnings, and informational messages. Software documentation is provided by an on-line context-sensitive help system. *Wizards* provide a means to quickly populate scenario settings for simulation, initial orbit determination, least squares estimation, and filtering. *Event control* gives an analyst the option of specifying appropriate output files or scripts following filter and/or simulator events, such as internal errors (*e.g.*, when a satellite has reached re-entry altitude), starting, stopping, resuming, interrupted halting, and ending of measurements.

ODTK provides a variety of reporting styles with customizable features. The *Static Product Builder* is an interface for creating, editing, and displaying reports and graphs, or exporting data to a variety of standard formats used by popular analysis software. The *Dynamic Product Selector* displays reports and graphs that continuously update during the filtering process so that filter performance can be visually monitored in real time.

ODTK provides a complete application programming interface (API) enabling automation and customized interfaces.

MEASUREMENT PROCESSING

Tracker Facilities

Facility locations are specified in the Earth-fixed frame, in a variety of coordinate variables. A precise station-motion model is implemented according to IERS Conventions, to include tectonic-plate motion, vertical motion due to Earth tides and coastal ocean-tide loading, and rotational deformation (or, pole tide).[3] A facility's location can be estimated by the filter/smoother, and a randomly deviated location can be simulated. All three (3) components of position can be estimated, or just the horizontal location assuming fixed altitude. Estimation of facility location is supported by various measurement types, within an *a priori* location uncertainty. Various statistical properties and measurement-processing attributes can be further set for the facility's measurement and tracking elements, discussed in the sequel.

GPS receivers, emitters, and/or transponders can be attached to a facility. The *GPS-receiver* object allows for simulation and/or processing of GPS measurements. The *emitter* object represents a source beacon from which one can simulate and/or process signal difference-of-arrival measurements. The *transponder* object allows for simulation and/or processing of two way measurements and multi-legged measurements involving relays (Figures 1 & 2).

Tracker Satellites

Orbit location may be specified as either Cartesian or Keplerian. The choices of inertial reference frames include the mean equator and equinox at J2000, or the true equator and equinox of date (Table 4). The Earth-centered, Earth-fixed (ECEF) frame is also available for the Cartesian input option. In ODTK, a satellite orbit can be estimated from tracking data, or it can be propagated from initial conditions, or it can be defined by an external ephemeris. An orbit uncertainty matrix is defined by the analyst to compute the satellite's initial filter covariance, and it is used by the simulator to randomly deviate the initial orbit state. The matrix elements represent an error-covariance in radial (R), in-track (I), and cross-track (C) position and velocity. The orbit uncertainty matrix can also specify the uncertainty of a satellite orbit defined by an external reference ephemeris; in this case, the orbital uncertainty is assumed constant in the RIC frame. Alternatively, the uncertainty of a satellite orbit defined by an external reference ephemeris can be provided as part of the reference ephemeris.

Satellite attitude specifications provide the means of computing the rotation between the inertial frame in which the equations of motion are employed and the body frame, and is used in the determination of antenna phase center from the center of mass of the satellite. Antenna phase center is specified relative to a body frame for the satellite.

Measurement Modeling

Stochastic Modeling and Process Noise. A key feature of ODTK is its ability to model stochastic process noise for state, acceleration, and measurement errors. Process noise is a mathematical model of errors in the system dynamics.[4] The stochastic process-noise model primarily adopted in ODTK is a stationary, two-parameter *Gauss-Markov sequence*, its scalar representation being:

$$x(t_{k+1}) = \Phi(t_{k+1}, t_k)\,x(t_k) + \sqrt{1-\Phi^2(t_{k+1}, t_k)}\,w(t_{k+1});\ \ k \in \{0,1,2,\mathbf{L}\};\ w \sim N(0, s_w^2), \quad (1)$$

where $\Phi(t_{k+1}, t_k) = e^{a|t_{k+1}-t_k|}$, s_w^2 is the variance of independent (white) Gaussian noise, and α is a constant < 0 prescribing the degree of process autocorrelation. In practice, the analyst defines α through the exponential half-life $\tau_{1/2} = \ln(1/2) / \alpha$.[*] The half-life regulates how quickly the bias value can be changed during prediction.

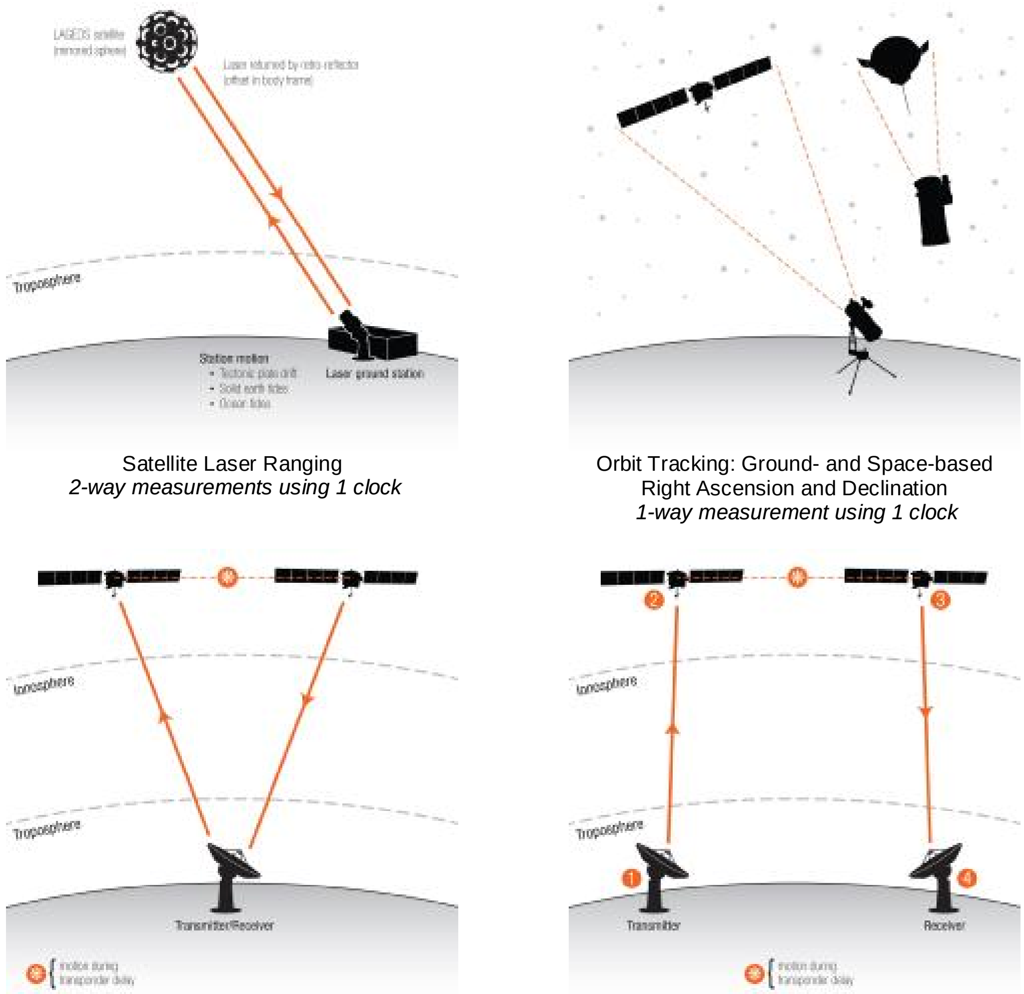
There are a couple of different process-noise approaches for filters.[5] *Dynamic-noise compensation* attempts to model unknown acceleration errors by assuming they can be characterized by the Langevin equation, a first-order linear stochastic differential equation. The general solution to the Langevin equation is Gauss-Markov and attributable to Ornstein and Uhlenbeck.[6] The model expressed in Eq. (1) is also Gauss-Markov, and related to the Ornstein-Uhlenbeck (O-U) process in that both demonstrate an exponentially fading autocorrelation function.

Table 2. MEASUREMENT STATISTICS

Property	Description
Bias	Constant offset between the value predicted by the measurement model and the observed value, defined in the sense that observed = predicted + bias.
Bias Half-Life	The half-life of the exponentially-decaying Gauss Markov bias.
Bias σ	Square root of the variance of the exponentially-decaying Gauss-Markov bias associated with a measurement model. The bias is used to estimate/simulate time varying biases associated with the measurement system.
White Noise σ	Square root of the variance representing the random uncertainty in the measurements.

[*] For a time interval equal to the half-life $\tau_{1/2}$, a Gauss Markov bias will decay by a factor of two (2) during estimation, in the absence of measurements.

Measurement Statistics. The parameters that control the level of process noise for simulation and estimation purposes are listed as *measurement statistics* belonging to the tracking object. The most significant measurement statistic settings are listed in Table 2.



Satellite Laser Ranging
2-way measurements using 1 clock

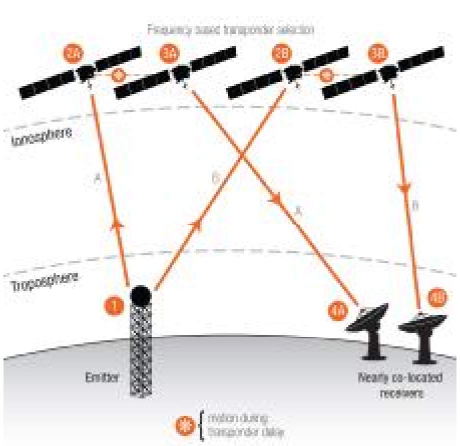
Orbit Tracking: Ground- and Space-based Right Ascension and Declination
1-way measurement using 1 clock

Ground-based Two-way Range
2-way measurements using 1 clock

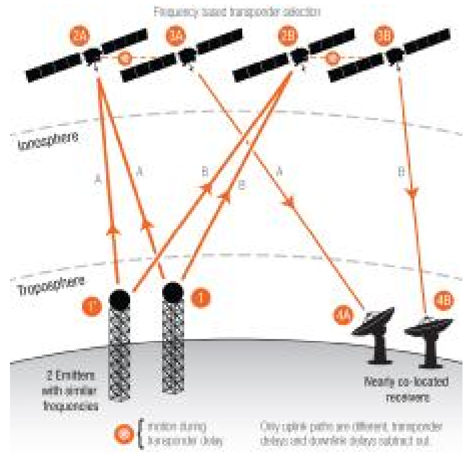
Ground-based Bistatic Range
1-way measurements with 2 synchronized clocks

Figure 1. Measurement Types Involving a Single Satellite

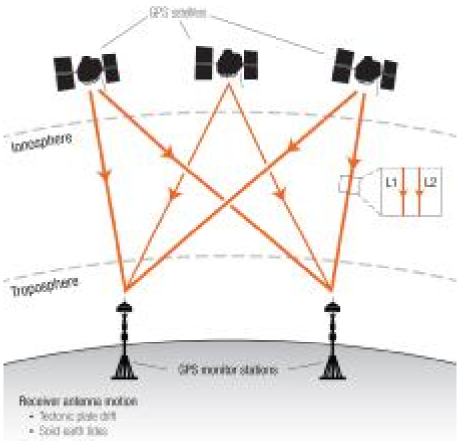
Measurement Types. Table 3 provides a listing of tracking data available in ODTK, while Figure 1 and Figure 2 illustrate them. Some measurement types are not collected directly by tracking devices, but are combinations of more basic measurements. They are not simulated directly in ODTK, but are constructed during the estimation process by the mathematical combination of the base measurements. Doubly-differenced (DD) GPS measurements can be created using an integrated tool from simulated base measurements. For many measurement types, biases can be estimated from the observations. If the biases are "temporal", they can be modeled as exponentially decaying Gauss-Markov processes as defined by Equation (1), otherwise they are treated as time constant.



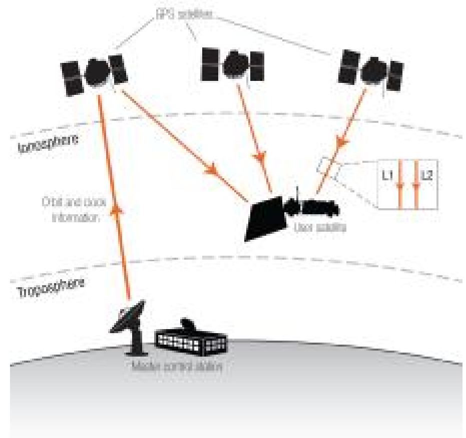
Time & Frequency Difference of Arrival (TDOA & FDOA)
1-way measurements using 1 clock



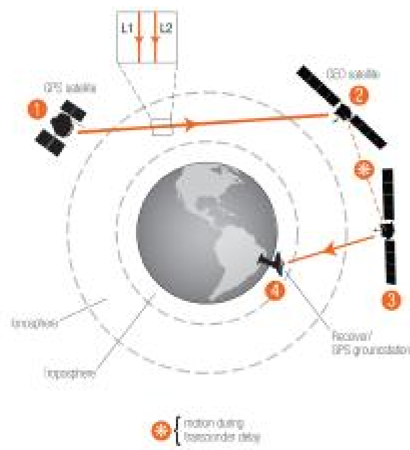
Singly Differenced TDOA and FDOA
1-way measurements using 1 clock



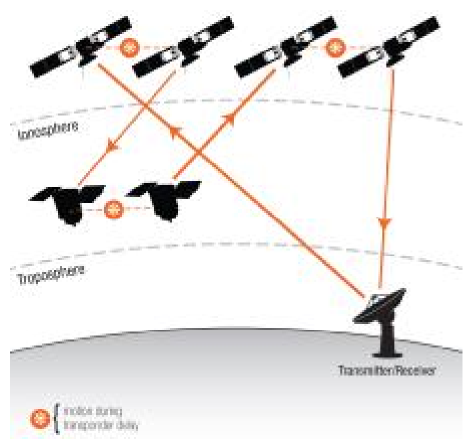
Ground-based GPS Pseudorange & Carrier Phase
1-way measurements using 2 clocks



GPS Pseudorange
1-way measurements using 2 clocks



Two-legged GPS Pseudorange
1-way measurements using 2 clocks



Four-legged Two-way Range
2-way measurements using 1 clock

Figure 2. Measurement Types Involving Multiple Satellites

Table 3. ODTK MEASUREMENT TYPES

Tracking Data Types	Modeled?	Simulated?	Estimate Bias?
Ground-based range, azimuth, elevation, Doppler	Yes	Yes	Temporal
Ground-based right ascension & declination	Yes	Yes	Temporal
Space-based range, azimuth, elevation	Yes	Yes	Temporal
Space-based right ascension & declination	Yes	Yes	Temporal
GPS pseudorange (CA, P1, P2)	Yes	Yes	Static
GPS pseudorange (dual frequency)	Yes	Constructed	Static
Singly-differenced (SD) GPS pseudorange (CA, P1, P2, DF)	Yes	Constructed	No
GPS carrier phase (LA, L1, L2)	Yes	Yes	Static
GPS carrier phase (dual frequency)	Yes	Constructed	Static
Singly-differenced GPS carrier phase (LA, L1, L2, DF)	Yes	Constructed	No
Doubly-differenced (DD) pseudorange (CA, P1, DF)	Yes	with tool	No
Doubly-differenced carrier phase (CA, L1, DF)	Yes	with tool	No
2-legged (transponded) GPS pseudorange (CA)	Yes	Yes	Temporal
GPS Navigation solution (CA, dual-frequency)	Yes	with tool	No
3-legged (1-way relayed) TDRS* Doppler	Yes	Yes	Temporal
4-legged (2-way relayed) TDRS range	Yes	Yes	Temporal
5-legged (2-way relayed) TDRS Doppler	Yes	Yes	Temporal
BRTS† Doppler	Yes	Yes	Temporal
BRTS range	Yes	Yes	Temporal
Bistatic range (one-way)	Yes	Yes	Temporal
Time-difference of arrival and rate (TDOA, TDOADot)	Yes	Yes	Temporal
Frequency-difference of arrival (FDOA)	Yes	Yes	No
Singly-Differenced TDOA, FDOA	Yes	Constructed	No
Space-based TDOA, TDOADot	Yes	Yes	Temporal
Space-based FDOA	Yes	Yes	No
Satellite laser ranging	Yes	Yes	Temporal
Space-to-ground Doppler (*e.g.* DORIS)	No	No	-
Multiple other tracking type (XY-angles, *etc.*)	No	No	-
Direction cosines	No	No	-
Altimetry / altimetric crossovers	No	No	-

Selection of pseudorange and phase-count measurement types has implications to simulator and filter processes if certain derived measurement are also chosen, because the derived measurements can be processed instead of the base measurements. This convenience allows an analyst to perform, for example, analyses whereby CA pseudorange measurements are simulated, and singly-differenced (SD) CA pseudorange measurements are processed. Both types can remain specified, without having to change

* TDRS(S) = Tracking and Data Relay Satellite (System)
† BRTS = Bilateral Ranging Transponder System

the selected measurement types. More complete descriptions of these measurement types are provided by Wright (2006).[7]

Frames of Reference. The analyst can specify the reference frame in which the right ascension and declination angles will be computed. This reference frame is used in the creation of simulated data, and in the processing of data when the tracking data file does not explicitly specify the observational frame of reference. Table 4 lists reference frames supported for optical-data processing.

Table 4. OPTICAL REFERENCE FRAMES[*]

Frame	Description
MEME B1950	Mean equator and mean equinox of B1950 reference frame, based on FK4.
MEME J2000	Mean equator and mean equinox of J2000 reference frame, based on FK5.
MEME of Date	Mean equator and mean equinox of the observation epoch, based on FK5.
TEME of Date	True equator and mean equinox of the observation epoch, based on FK5.
TETE of Date	True equator and true equinox of the observation epoch, based on FK5.
TETE of Jan0	True equator and true equinox of January 0 of the year of the observation, based on FK5.

Measurement Delimiting. An analyst may delimit the available measurements, satellites, trackers, and emitters for use in each computational object (simulator, filter, *etc*.). For example, there is a *Measurement Types* property for each satellite that lists the allowable measurement models that may be used to track the satellite, including those associated with an attached on-board GPS receiver. Measurements types not included in this list will be discarded during the estimation process.

Tracking Strands. A tracking strand is a sequence of objects in ODTK, specified in an order dictated by the measurement models, required for the modeling of an observation. The tracking strand concept supports more complicated tracking systems such as TDRSS, where relay satellites and relay ground stations are involved in the modeling of a single observation. The list of available tracking strands is based on the trackers in the scenario and the list of measurement models associated with each tracker. Delimiting based on tracking strands is available for the simulation of measurements.

Ionosphere Model. An ionosphere model can be applied when a satellite is tracked at a frequency that is significantly affected by the ionosphere (*e.g.*, GPS CA). The magnitude of the ionosphere range delay is inversely proportional to the square of the frequency, thus, a higher frequency measurement yields a smaller ionosphere delay. Currently, the International Reference Ionosphere (2001) is the only available ionosphere model.[8] The IRI models are self-contained; however, they require current IRI model data. ODTK provides a utility to update data files, including the IRI model data files.

[*] The equator and equinox are those conventionally defined by the IAU 1976 Theory of Precession and IAU 1980 Theory of Nutation.

Troposphere Model. Meteorological data, consisting of the pressure, temperature, and humidity at the facility, are needed to support computations with the ***Marini-Murray model*** or ***Saastamoinen model***. These data may be specified as constant values, in tables via text files, or may be extracted from the input tracking data. Otherwise, the ***SCF model*** only requires that the analyst specify nominal surface refractivity at each facility location, which can be modeled as a polynomial function from zeroth- to tenth-order spanning one year. Given sufficient tracking, a time-varying correction can be estimated to the wet component of a facility's ***a priori*** tropospheric zenith delay.

Ranging Method. Ranging-method options reflect the difference between cooperative and surveillance tracking. ***Transponder ranging*** is cooperative, indicating that a tracking station that generates tracking data only through an active transponder on the spacecraft (***e.g.***, Space-Ground Link System). ***Skin-track ranging*** is passive surveillance, indicating a radar or optical tracking station that does not actively use a satellite transponder. For this latter option, the transponder bias is unused for the range and Doppler measurements.

Measurement Editing

Within ODTK's Kalman filter measurement-update algorithm, a "predicted residual" δy is computed when an observed measurement is subtracted by a computed measurement based on the predicted state.* The root variance $\sigma_{\delta y}$ of residual δy can be computed from the combined uncertainty of the predicted state and the observation error, as expressed by their respective (co)variances. Under the operating assumptions of the filter, the standardized-residual test statistic, $\delta y / \sigma_{\delta y}$, is distributed as standard normal, $N(0, 1)$.

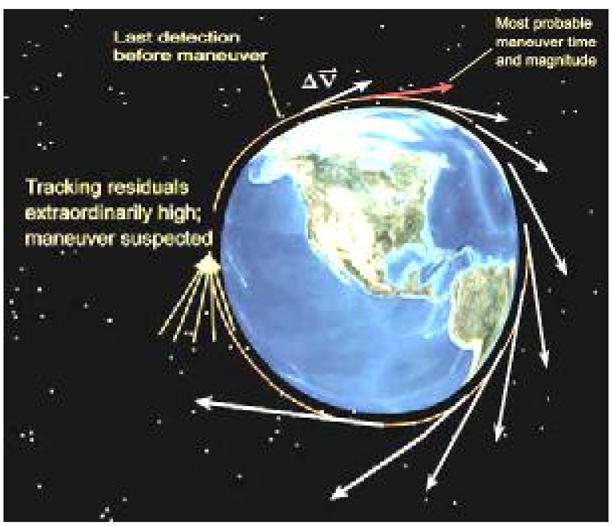


Figure 3. Maneuver Detection via Large Residuals

It is improbable that a single value of $|\,\delta y / \sigma_{\delta y}\,|$ would be greater than say, three (3), owing to chance. Generally, if $|\,\delta y / \sigma_{\delta y}\,|$ exceeds such a threshold, the measurement will be ignored by the filter. A customized ***dynamic editor*** is also provided to address special cases, such as bad data during filter initialization, or apparently bad data after a maneuver (Figure 3).

Tracking data may be ignored by the filter for a variety of other reasons, including sensor elevation constraints, an invalid tracker, the angle is too close to pole (90º of right ascension or elevation), the GPS receiver signal-to-noise is too low, or an invalid noise value is imported from tracking data. The low-elevation constraint is a useful means for dealing with GPS multi-path errors, for example. There are also various

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* In filter-smoother applications, the term ***residual*** is somewhat ambiguous. A residual based on the predicted, or pre-fit, state, is sometimes known as an ***innovation***. The label implies the imminent introduction of new information that may change the estimate, and distinguishes it from the post-fit residual commonly seen in least-squares applications.

error checks for multi-legged measurements such as TDRS and BTDS. A variety of other input data manipulation and data editing techniques are provided to further address the vagaries of processing real data.

FORCE MODELING

ODTK uses a Variation of Parameters (VOP) formulation of the equations of motion, coupled with several single-step numerical integrators. The available force modeling includes spherical-harmonic gravity, solid-Earth and ocean tides, drag due to one of several different atmospheric density models, solar- and Earth-radiation pressure, maneuver thrusting (instant and finite duration), relativistic perturbations, and the gravitational effects of the Sun, Moon, and major planets (Table 5).

Table 5. ODTK FORCE MODEL CAPABILITY

Force Modeling	Modeled?	Simulated?	Estimated?
Earth gravity field, solid earth tides	Yes	No	No
Ocean tide perturbations	Yes	No	No
Third body gravity (Solar, lunar, planetary)	Yes	No	No
Atmospheric density (air drag)	Yes	Yes	Temporal
Drag coefficient	Yes	Yes	Temporal
Solar-radiation pressure	Yes	Yes	Temporal
Earth albedo	Yes	Yes	Temporal
Impulsive thrusting	Yes	Yes	Static
Finite-duration maneuvers	Yes	Yes	Temporal
Custom accelerations (plug-ins)	Yes	Yes	No

Gravity

The gravity model for ODTK is provided via text files. The analyst may specify the maximum degree and order to be used. Perturbing accelerations due to solid and/or ocean tides follow IERS Conventions (2003).[3] Tide models can also be truncated to speed processing. ODTK's implementation of General Relativity effects (Schwarzschild solution, geodesic and Lense–Thirring precessions) also follows IERS standards.

ODTK's gravity process noise accounts for certain errors associated with integrating through the gravity field.[1] Gravity-field process noise can be always used, always ignored, or dependent upon the satellite's orbit and other pre-defined factors. The gravity process noise model allows the analyst to account for errors of commission (uncertainty in the gravity field coefficients) and/or errors of omission (uncertainty due to truncating the gravity field). Adding gravity process noise increases the orbit covariance. Third-body accelerations due to the Sun, Moon, and/or major planets are available.

Air Drag

Deceleration due to atmospheric drag can be automatically activated in ODTK when the perigee of the spacecraft's orbit falls below a minimum perigee-altitude threshold defined by the analyst. Available atmospheric models include CIRA 1972 (an empirical model of atmospheric temperature and densities as recommended by COSPAR, and the same as Jacchia 1971), Jacchia-Roberts (similar to Jacchia 1971 but uses analytical methods to improve throughput), MSISE 1990 (an empirical constituent density model valid from zero to 1000 km altitude, based on satellite data), and NRLMSISE 2000 (an empirical constituent density model more recently developed by the US Naval Research Laboratory, also based on satellite data). In ODTK, satellite ballistic coefficient is defined as $B = C_d A / m$, where C_d is coefficient of drag, A is frontal cross-sectional area, and m is the satellite mass; an analyst may choose to work with B directly, or input the individual components C_d , A, and m if those are known individually.

A novel feature of ODTK is its ability to estimate a relative correction to the ballistic coefficient while simultaneously estimating a relative correction to the local atmospheric density.[9, 10] The relative corrections to atmospheric density and ballistic coefficient are each modeled as Gauss-Markov processes, wherein the analyst can define their exponential half-lives. The two states become separable when the half-life of the density correction is significantly different from the half-life of the B correction and when the atmosphere is in an excited state. Except during an initialization period, the root-variance of the density correction is calculated internally from the atmospheric model, while the root-variance of the B correction is analyst defined.

Radiation Pressure

Solar-radiation pressure (SRP) is modeled using a penumbral cone model for the Earth's shadow and the Moon's shadow.[11] The SRP model can be activated automatically when the spacecraft's orbit apogee crosses a threshold value. A relative correction to the satellite's nominal solar pressure coefficient can be estimated by the filter, modeled as an exponentially decaying Gauss-Markov process, where the analyst can define the exponential half-life of the process. A spherical radiation model is provided and can be configured as either diffusely reflecting or perfectly absorbing.[12] ODTK can account for the change in solar pressure if a satellite is in the lunar shadow, a recommended option for high-accuracy satellite programs. White process noise can be further added to the in-ecliptic and out-of-ecliptic directions normal to the Sun-to-satellite vector. This is useful should significant solar pressure accelerations exist in these directions, as solar pressure ordinarily accounts for acceleration along the satellite-to-Sun direction.

An Earth albedo model (acceleration due to sunlight reflected from Earth) and a thermal radiation pressure model (acceleration due to black-body radiation from Earth) are both available. These computationally intensive models are defined by an analyst-selectable ground-reflection file; tradeoffs in computation versus accuracy can be controlled by varying the grid sampling of the file.

GPS Solar Pressure Models & Parameters

The GPS satellite solar-pressure models are specific to the GPS Block types (II, IIA, IIR, IIRM, *etc.*). JPL's Block IIA and Block IIR models are recommended for precision work; these are the GSPM.04a models derived from a study of non-eclipsing satellites, and the GSPM.04ae extension for satellites undergoing eclipse.[13, 14] Optional legacy models include the Aerospace T20 (Block IIA) and the Aerospace T30 (Block

IIR) models.[15, 16] ODTK's implementations employ two scale parameters: one primarily affects acceleration in the satellite-body-fixed X-Z plane containing the Sun-to-satellite line (K_1), and the other primarily affects acceleration along the body-fixed Y axis of the satellite (K_2). These unitless scale factors can be estimated and are modeled as exponentially decaying Gauss-Markov processes, where the analyst defines the process variances and exponential half-lives. Prior values for K_1 and K_2 are not generally available and must be inferred from the tracking data or precision ephemerides. A process that generates reasonable values for K_1 and K_2 using the publicly available SP3 files is documented as part of the ODTK Help System.

Maneuvers

So-called *instant maneuvers* are impulsive ΔV-type events applied at a single time, where the time and the change in velocity are computed with algorithms given in Wright (2006).[7] ODTK makes available four types of instant maneuvers: *velocity change* (a physically non-realistic method adequate for short-term thrusting that treats the ΔV as instantaneous), *acceleration history* (determined assuming time history of satellite acceleration during the maneuver), *constant thrust* (assumes increasing acceleration with decreasing mass per $a(t) = F_{const} / m(t)$), and *constant acceleration* (assumes decreasing force with decreasing mass per $F(t) = m(t) \cdot a_{const}$). Since an interval of thrusting is approximated as an impulse, any tracking data collected during the thrust interval may be mismodeled. A data-exclusion interval can be specified to ignore any such tracking data.

So-called *finite [time-interval] maneuvers* are also provided for accurate modeling of long-duration thrusting.[17] The finite maneuver option only employs the *acceleration history* and *constant thrust* methods integrated over the duration of the maneuver. Multiple-impulsive or finite maneuvers can be specified on a single satellite or multiple satellites. When multiple finite maneuvers are specified for a single satellite, the maneuvers can be configured to be *independent* (*i.e*., each maneuver is assigned its own estimation states - appropriate if there is a long time between the maneuvers, or, if the maneuvers use different thrusters). The maneuvers can also be configured to be *related* (multiple maneuvers share the same estimation states - appropriate when there is a short time between the maneuvers and common thrusters are used).

Given sufficient tracking data, the magnitude and/or direction of a finite-maneuver can be estimated by the filter, each component being modeled as an exponentially decaying Gauss-Markov process, where the analyst can define the exponential half-life of the process. Estimates of impulsive maneuvers can be extracted from the output of the smoother in a manner described by Woodburn *et al*. (2003).[18] Satellite maneuvers can be designated relative to the *Gaussian* frame (radial, transverse, cross-track, or RIC), *Frenet* frame (radial, tangential, cross-track), or *inertial* frame, and specified by either Cartesian components (X, Y, Z) or angular pointing direction and magnitude. The analyst may further specify ODTK's *unmodeled accelerations* option to add process noise in the radial, in-track, and cross-track directions, to account for unmodeled phenomena such as out-gassing. The process noise is applied to the filtered covariance at each time update.

Analyst (Plug-in) Force Models

A single force-model plug-in point is provided for the customization of the accelerations affecting satellite propagation. A typical use might be to implement a specialized non-conservative force model, such as a box-wing model for solar pressure or drag. Plug-ins may be written in a number of languages, such as Perl, VB Script, C++, C#, and Visual Basic .NET. Details regarding the ODTK Plugging option are found through the ODTK Help System.

SIMULATOR

The purpose of the simulator is to create a set of realistically deviated measurements by varying the initial conditions of satellites and measurement biases, and by adding noise to modeled measurements. The analyst controls where deviations are applied, and if no deviations are applied, the resultant simulated measurements should provide nearly zero residuals when processed by the filter. The analyst can opt to apply a scale factor to the deviations that effectively acts as a σ-multiplier.

A parameter is "deviated" when its value is perturbed by a Gaussian pseudo-random number drawn from a $N(0, \sigma)$ distribution. The deviated result defines a randomly defined "truth" for the simulation. Options include deviating orbits (the initial satellite orbit state is deviated against the full orbit covariance in the RIC frame), atmospheric density, ballistic coefficient, solar-pressure coefficient, transponder delay, measurement biases, troposphere biases, maneuvers, facility locations, and antenna locations. White noise can be added to simulated measurements, and simulated process noise can be further added to density corrections, solar pressure, transponder delays, measurement biases, direction and magnitude of finite maneuvers, and GPS receiver clocks.[*] The sequences for simulated process noise, except for clocks, are described by Equation 1. Details of the clock process noise model are found in Wright (2006).[7] *Custom tracking intervals* allow the analyst to define inclusion and exclusion intervals and vary time step by facility and/or satellite.

ORBIT DETERMINATION

Initial Orbit Determination

Initial orbit determination (IOD) creates position and velocity from satellite tracking data, when no prior orbit is available. IOD methods assume two-body dynamics and the results are quite sensitive to observation errors. An analyst can employ different IOD methods, depending upon the tracking data available. The *Herrick-Gibbs* method requires three (3) sets of range, azimuth, and elevation at different times. *Gooding's angles-only* method requires at least three (3) sets of angles (ground-based azimuth and elevation, ground-based R.A. and declination, or space-based R.A. and declination).[19, 20] Gooding's method accepts measurements of the same satellite from multiple trackers.

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[*] The stochastic error model applied to the GPS Receiver(s) clock is valid for crystal clocks and atomic clocks. The model is a superposition of errors to frequency white noise, frequency random walk, and aging. However, frequency flicker noise is not modeled.

A third IOD option is the *GPS navigation solution* method.[*] This method uses a series of RINEX-formatted GPS observations as inputs. The method estimates a spacecraft's position and clock-phase using an analytical navigation solution algorithm by Yang and Chen for several points around the time of interest, then fits of an interpolating polynomial through those positions and uses the derivative to estimate the velocity.[21] Measurements from at least four (4) GPS constellation space vehicles are necessary at each time, including the solution epoch. ODTK's *IOD Wizard* is available to aid with the IOD process.

Table 6. ODTK ESTIMATION CAPABILITY

Estimation Capability	ODTK method
Solve for orbit (position and velocity)	Herrick-Gibbs IOD Gooding angles-only IOD GPS navigation solution IOD Weighted least-squares estimator Optimal Filter / Smoother
Solve for orbit, ballistic (air drag) & solar-pressure coefficients	Weighted least-squares estimator Optimal Filter / Smoother (OF/S)
Solve for correction to atmospheric density	OF/S
Solve for corrections to maneuvers	OF/S
Simultaneous orbit determination on multiple satellites	OF/S
Solve for GPS receiver clock phase and drift	OF/S
Solve for facility locations (2-D or 3-D)	OF/S
Model spacecraft attitude and antenna offsets	OF/S
Solve for antenna phase center	OF/S
Solve for measurement biases	OF/S - see Table 3
Provide realistic covariance	OF/S

Least Squares

Weighted least-squares estimators are the oldest and most well known for orbit determination purposes. ODTK employs a weighted least-squares estimator to refine the orbit derived from the IOD method. The batch-least-squares process is limited in the number of states estimated. These include position, velocity, solar-radiation-pressure correction, and ballistic-coefficient correction (Table 6).

Optimal Filter

ODTK employs a state-of-the-art optimal, current-time sequential filter.[22] The Kalman filter measurement update is used. The time-update equations differ from those of a Kalman filter due to the use of process-noise models based on the uncertainty of the underlying physical models. This enables the calculation and propagation of a more realistic state error covariance. The optimum filter can also estimate time-varying biases

[*] GPS navigation solutions are generated from the pseudorange measurements and represent an instantaneous estimate of the receiver position.

on the tracking system as well as time-varying corrections to the force models.[*] A realistic covariance has a variety of uses, including system performance analyses, optimization of tasking, and calculation of the probability of collision. As discussed, the realistic state error covariance further enables the autonomous identification and editing of outlying measurement residuals. The ODTK filter provides various output control for data archiving, display, output for the smoother, and creation of STK ephemeris files for display and file operations.

State error Transition Method. The state error transition matrix is an *n* x *n* matrix that maps a slight deviation in the states at one epoch into a deviation at another epoch.[†] There are two methods of orbit error transition in ODTK: *variational equations*, and *numerical partial derivatives*. The variational equations are the partial derivatives of the accelerations with respect to the satellite position and velocity, and are used to compute the time derivative of the orbit error transition matrix. When using variational equations, the orbit error transition matrix is numerically integrated with the satellite. This method is efficient and works well for both dense tracking data and prediction.

When using the numerical partial derivatives method of transition, ODTK propagates six additional trajectories (each one originating from a small deviation having been applied to each component of the position or velocity). Trajectory differences are used to numerically approximate the partial derivatives that contribute to the orbit error transition matrix. This method is comparatively slow, but useful for comparison purposes. The following numerical integrators can be set for each satellite: Bulirsch-Stoer integration (based on Richardson extrapolation with automatic step size control), fourth-order Runge-Kutta (no step-size error control), and seventh-order Runge-Kutta-Fehlberg with eighth-order step-size error control.

Filter Restart. The sequential filter is initialized with an initial epoch, initial state estimate, initial state-estimate-error covariance matrix, measurement data to update the state estimate, and all support data and model parameters to operate the filter process. ODTK also provides an automatic *filter restart* capability that periodically saves this necessary information to a “restart disk file” at intervals specified by the analyst. The filter process can then be restarted from any epoch in the restart file as required.

Initialization of the filter from the most recent restart record is the normal processing mode for operational orbit determination. Use of the restart capability also allows the filter to resume operation at an earlier epoch without having to reprocess historical data and without loss of accuracy or information. If the filter diverges during a long computer run during operations (***e.g.***, the analyst neglected to model an orbit maneuver), use of selectable restart epochs can expedite recovery.

Flexible State Space. ODTK allows an analyst to add, remove, or change elements within the filter state space without requiring re-initialization of the filter. Table 7 lists several use cases supported as flexible-state-space options. Generally an analyst requests one or several of these actions by changing the scenario via the GUI and restarting the filter.

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[*] There is no limit to the maximum size of ODTK’s filter state.
[†] The orbit error transition matrix is part of the overall state error transition matrix, which includes all elements of the estimated state. The state error transition matrix is used in the propagation of the covariance matrix.

Table 7. USE CASES SUPPORTED AS FLEXIBLE-STATE-SPACE OPTIONS

Add or delete a satellite	Add or delete a measurement type or change its noise characteristics
Add or delete a ground facility	Change the estimated state dimension (*e.g.*, add or delete estimation of drag or radiation pressure coefficients, or biases)
Replace / update satellite position and velocity	Change the state noise statistics (*e.g.*, the exponential half-life or standard deviation of drag or radiation pressure coefficients, or biases)
Add or delete finite maneuver events	Change the bias and corresponding noise characteristics for facilities or transponders

When a state is added or changed, the cross-correlations to other states are initialized as zero under the assumption that the new content is independent of existing content. Because flexible-state-space actions change the dimension and content of the filter state, the analyst can be required to take additional actions to save and use intermediate files, such as restart files, smoother-input files, and output files that drive reports and graphs.

SMOOTHER

ODTK's fixed-interval smoother combines filtered state and covariance information in reverse chronological order to calculate the best post-fit estimate of the orbit throughout any interval of interest. The advantages of the smoother are a significantly smaller covariance than the filter covariance (Figure 4), and state estimates that behave smoothly.

ODTK's fixed-interval sequential smoother is a non-linear adaptation of the linear fixed-interval optimal smoother presented by Meditch (1969).[23] The smoother operates using outputs saved from the sequential filter, and the VOP trajectory propagator runs backwards with time to accommodate the smoother's state-transition function. As the smoother runs backwards through time, it smoothes ephemeris discontinuities caused by state corrections due to processed measurements (*i.e.*, there is no improvement in the smoothed result until a measurement update is encountered). Smoothing operations can be applied over the results of multiple filter runs to provide longer duration smoothed outputs. A mathematical overview of the ODTK smoother is provided by Wright (2006), and a detailed description is available via the ODTK Help System.[7]

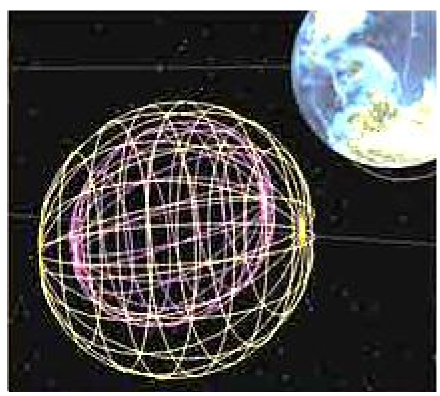

Figure 4. Smoother Covariance v. Larger Filter Covariance

Filter-Smoother Consistency Test

The *filter-smoother consistency test* is useful for general model validation.[24] McReynolds (1984) proved that the difference between the filtered state and smoother state is normally distributed in *n* dimensions (assuming *n* is the size of the state-difference vector). He also showed that the variances and correlations of the state-difference vector are equal to the filter covariance subtracted from the smoother covariance. Plotting this difference vector over time creates a population that should be normal. If it is abnormal, then this is interpreted as a defect in the filter-smoother model.

ODTK provides a graphical method of examining filter-smoother consistency of each state element, instead of a full multivariate test statistic originally proposed by McReynolds. For the single-state case, the filter-smoother state differences are divided by the filter-smoother variance differences. Filter-smoother consistency is generally claimed when this metric stays within ± 3 over the fit interval. This single-element test provides the analyst with very useful diagnostic information in the presence of erroneous filter inputs or model specifications.

GLOBAL POSITIONING SYSTEM

ODTK supports simultaneous orbit and clock estimation of the entire GPS satellite navigation system. An analyst can process live monitor station pseudorange and phase data to solve for, and otherwise perform analysis on the GPS SVs,[*] their clocks, and the monitor stations supporting the GPS navigation mission. In addition to conventional orbit determination, clock estimation and modeling are required. The GPS orbit is relatively straightforward to estimate, with the greatest error sources due to solar pressure, eclipsing, and infrequent maneuvers. However, the clock estimation problem is complicated, with dozens of clocks all behaving differently, aging independently, suffering from clock events (discussed in the sequel), and occasionally being replaced.

ODTK's *GPS Constellation* object can be initialized from two text files: an AGI-supplied *reference catalog file* that initializes PRN-dependent model parameters (including clock data, group delay, antenna offsets, beam field-of-view, and active / inactive status), and externally-supplied *reference source files* that provide the satellite orbit and clock states as a function of time. Reference source files are available through various public agencies (such as NGA's SP3 files) and can be used to initialize orbit and clock states are for all satellites in the constellation. If monitor station tracking data are available, the GPS SVs' orbit states and clock states may be estimated entirely from that data.

GPS monitor stations are represented in ODTK as facilities with GPS receivers. As such, they benefit from the precision Earth modeling. The locations of antenna / receiver objects are Cartesian offsets relative to the facility location, sometimes called *eccentricities*. If a GPS receiver has multiple antennas (such as a primary and a secondary), ODTK can switch between antennas automatically if an antenna identification number (ID) is included with the monitor station data.

ODTK can process GPS pseudorange and phase count measurements from the GPS receivers, and supports standard RINEX formats for GPS data. Other critical data, required to support orbit and clock state estimation, are not publicly available, such as clock-phase and frequency offsets for monitor stations, process-noise settings for various clocks, solar-pressure coefficients for the GPS SVs and solar-pressure process noise

[*] *Global Positioning System Space Vehicles, i.e.*, satellites broadcasting as part of the GPS constellation.

settings. These data have to be derived before an ODTK scenario is properly defined. Various ODTK processing options will assist the analyst in estimating these parameters:

- solve for monitor station clock states while using an SP3 reference for SV ephemeris and clocks,
- solve for GPS SV orbits, using the SP3 reference to define the SV clocks,
- solve for GPS SV clocks, using the SP3 reference to define the SV orbits,
- solve for all state parameters simultaneously,
- eliminate monitor station clocks using single differencing,
- eliminate monitor station clocks and GPS SV clocks using double differencing, and
- process SP3 reference data (Earth-centered, Earth-fixed positions) as measurements.

GPS Clocks

Time indicated by a closed system of clocks, such as GPS, could drift with respect to an external standard. In order to support many high-accuracy GPS applications, it has become desirable to steer GPS time to maintain a constant offset in phase with UTC to within a few nanoseconds. The method that is used by the GPS Master Control Station involves a *clock steering* algorithm - a damped bang-bang controller - applied to the output of a ***composite ensemble*** composed of the most reliable clocks.[25] These algorithms are implemented in ODTK, controlled by attributes within the GPS Constellation object.

Clock behavior is defined by initial biases in ***phase*** (current time), ***frequency*** (derivative of phase), and ***aging*** (derivative of frequency). The clock models are extremely sensitive to these initial quantities. Phase and frequency biases can be estimated over time, given their initial root-variance uncertainties.[*] The aging σ specifies the uncertainty in the known long term trend in frequency. The clock stochastic error model parameter, a_0, prescribes the level of process noise associated with phase error due to frequency white noise, while a_{-2} is a stochastic error model parameter that prescribes the level of process noise associated with phase error due to frequency random walk. An Allan Variance diagram can aid the analyst can in adjusting the values of a_0, and a_{-2}.[26, 27]

A *clock event* is a disruption in an otherwise uniform measure of time. Such events can take many forms, for example, NGA monitor stations regularly reset their clocks, creating millisecond-level steps in clock phase. More serious perturbations to time readings occur when frequency standards are replaced, causing instantaneous offsets in phase, frequency, and drift. Clock events are detectable in pseudorange residuals and are therefore observable. In ODTK, clock events are treated analogous to impulsive maneuvers, where the analyst enters an approximate magnitude and uncertainty of the event (as observed from residuals). The analyst can control phase, frequency and aging separately for each GPS SV and each receiver.

Occasionally, an advisory is issued to ignore one or more of the GPS SVs for various reasons, including maneuvers, clock replacement, and satellite anomalies. At the Master Control Station the satellite is partitioned from the rest of the constellation to prevent any degradation in the navigation mission. ODTK supports this process by defining a separate partition in the filter to maintain the orbit(s) and estimate clock

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[*] Orbiting clocks experience secular rates due to relativity that can be corrected deterministically.

corrections for the off-line satellite(s). This partition still references monitor station-clock and troposphere estimates based on the remaining on-line GPS-constellation satellites, but it does not push any corrective information from the partitioned satellite(s) into estimates affecting the rest of the system.

Maintenance of a global navigation satellite system is perhaps the most complicated application of ODTK. A tutorial provides an overview of and guide to the application of ODTK to GPS satellite and clock estimation.[28]

CONCLUDING STATEMENT

The *Orbit Determination Tool Kit* is a commercial off-the-shelf software product, presently used by agencies and contractors for the analysis, design, development, and operation of various satellite systems. It has evolved into a robust tool capable of supporting the accuracy and timeliness requirements of most geocentric satellite missions. This was accomplished by conforming to international standards, by leveraging proven algorithms and models, by pursuing state-of-the-art information in open literature, and by continual technical refinement based on analysts' experience and feedback. This paper describes many of the features that have made ODTK functional for the applications it serves, and new features will continue to be added based on consumer requests. There are future plans to support interplanetary trajectory estimation, refine GPS capabilities, add more robust geolocation capabilities, and expand the GPS functionality to support the GALILEO and GLONASS global navigation systems.

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